Comparative Advantages, Trade and Endogenous Borders

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Abstract

We construct a model in which the equilibrium location of borders is determined by the forces of comparative advantage. Locations positioned along a line-shaped world have relative endowments as dictated by geography. Borders are locations that are costly for trade to go through, and which delimit countries within which trade is costless. We fix the number of borders, and investigate whether any subdivision is robust to a coalition of adjacent locations co-ordinating to redraw borders. In equilibrium, trade considerations rule out the existence of relatively large countries. This is because if such countries existed, neighbouring locations would like to be annexed by them, in order to access greater diversity and obtain greater gains from trade. Simulations show that higher heterogeneity of relative endowments—i.e., stronger comparative advantage forces—across space results in, on average, countries with more homogenous size. This prediction is confirmed by preliminary empirical analysis relating ecological diversity to heterogeneity of country size.

1 Introduction

The role of trade in shaping borders and nations has long been lingering in the mind of policy makers and social scientists. This is evident in the mercantilistic mindset of the 17th, 18th and late 19th centuries, when colonial expansion was justified on the assumption that "trade follows the flag": overseas trade would only flourish if overseas partners could be annexed to form a larger political entity. While colonization gave way to nation states, trade still affects territorial disputes. Several contemporary military conflicts, such as that between Ukraine and Russia, and sovereignty proposals, such as that in Scotland following Brexit, are presumably related to disagreements on spheres of free trade.[1]

In their seminal work, Alesina, Spolaore, and Wacziarg (2000) have investigated the optimal size of nations in a symmetric trade model where agents trade off trade costs (which are lower within countries than across countries) for the cost of cultural heterogeneity (which is larger in larger countries).[2] The central result is that the optimal number of countries should be smaller (and countries should be bigger) when cross-country trade costs are higher, relative to within-country trade costs. A similar result obtains in Casella (2001).

Although this literature offers important insights, it does not take into account some of the important links between trade and political borders. Consider for example the secession of the Latin America colonies from their colonial masters, Spain and Portugal, in the early 19th century. One key factor underpinning the secession was an ongoing change in trade *patterns*, due to the Industrial Revolution in Britain, which made it more attractive for raw materials-abundant Latin America to be able to trade freely with an increasingly manufactures-abundant rest of the world, as opposed to being constrained within an increasingly raw materials-abundant empire (Bonfatti, 2017). In other words, it was a change in the distribution of *comparative advantage* across the

[1] Ukraine signed a free trade agreements with the EU in 2014, despite pressure by Russia on Ukraine to join the Eurasian Customs Union. According to the Economist (2014), Russia's attempts "to snub a corresponding agreement with the EU helped inspire the Euromaidan protests that brought down Viktor Yanukovych's government and eventually spun into the strife that is now roiling the east of Ukraine." On a more peaceful venue, Scotland's 32 electoral regions voted all for remaining in the EU in the June 2016 Brexit referendum. In the aftermath, pro-independence voices in Scotland brought the issue of a secession vote from the UK back to the forefront of policy (BBC, 2017).

[2] A related paper is Alesina and Spolaore (1997), which however puts less emphasis on trade and more on the role of countries in reducing the cost of public good provision.

world which led to the re-drawing of political borders. Because they only consider trade between symmetric locations, Alesina, Spolaore, and Wacziarg (2000) cannot easily account for events such as this.

In this paper, we study the link between trade and endogenous borders in a model of comparative advantages. We consider a line-shaped world in which all locations have an equal endowment of "food", while endowments of "manufactures" increase linearly from west to east. If trade is allowed between locations, western locations have, relative to eastern locations, a comparative advantage in food. Countries, or sets of adjacent locations, can emerge as a way to reduce transactions costs between locations: for simplicity, we assume that trade is costless within countries, but prohibitively costly across countries. A subdivision of the world into countries is an equilibrium if and only if there exist no coalition of adjacent locations for which a deviation to a different subdivision (e.g. through secession) is a Pareto improvement.

Unlike in Alesina, Spolaore, and Wacziarg (2000), in our framework, bigger countries bring efficiency gains (due to lower transaction costs) but no efficiency costs. Still, the world may be politically fragmented in equilibrium. For example, *any* subdivision of the world in equally-sized countries can be an equilibrium. This is because different locations have different comparative advantages, and thus different preferences over a re-drawing of borders. For example, the merger of country A with a relatively food-abundant country B will generate more trade opportunities and thus an efficiency gain; however the terms of trade effect will also damage locations that are relatively food-abundant within A, as well as locations that are relatively food-scarce within B. These locations will then block the merger. A key assumption for this result is the absence of compensating transfers.

In comparative statics analysis, we relate the characteristics of the equilibria to the slope of the distribution of manufactures. Such slope is a measure of heterogeneity across locations, or of the strength of comparative advantage forces. Because the model admits infinite equilibria, we derive a large number of equilibria and study how their average characteristics relate to this parameter. We find that the variance of country sizes decreases in the spatial heterogeneity of endowments. The intuition for this result is the trade-off between size and heterogeneity in our model: the gains from trade for border locations increase with both the size of the country and the amount of heterogeneity. As heterogeneity increases, any pair of locations at a given distance reap more gains from trading with each other, so the need for being within the same country with more distant locations diminishes. As the largest country shrinks, so does the variance of country sizes.

Our paper relates to an empirical literature that links ecological diversity, and the trade opportunities that this generates, to the emergence of centralised states in Africa. Bates (1983) argues that centralised states were more likely to emerge in ecologically diverse areas, where their trade-supporting role was more valuable. Fenske (2014) tests this hypothesis using data on African ethnic groups, and finds a strong positive effect of ecological diversity on state centralisation. Differently from this literature, which takes the territories of African states as given, we investigate the effect of diversity on the endogenous location of borders. Michalopoulos (2012) investigates the geographic determinants of ethno-linguistic diversity, which is intrinsically related to the size and shape of countries. In recent work, Kitamura and Lagerlof (2016) examine how geography affects the location of borders between European states while Li and Zhang (2016) build an equilibrium model of trade and national borders to analyze how changes in trade costs drive border changes.

Our paper also relates to the literature on regional economic integration. In particular, Riezman (1985) focuses on coalitions of countries forming customs union. Similar to our setting, countries in a customs union eliminate trade barriers among themselves while they institute common trade barriers to the rest of the world. Riezman (1985) analyzes the stable core of such coalition formation with three countries. Instead of discrete countries, we consider locations in a continuous geography. By not allowing non-contiguous coalitions, geography imposes constraints on country formation, which in turn enables us to characterize a stable equilibrium that is robust to deviations by coalitions.

The next section describes the economic environment and presents intermediate results on the pattern of trade. Section 3 describes the mechanisms through which coalitions of locations can change borders. Our main results on the characterization of equilibrium borders are presented in Section 4. In Section 5, we rely on simulations to get further insights into the mechanisms and present comparative statics, which we take to the data in Section 6.

2 Economic Environment

2.1 Preferences and Endowments

There are two types of goods, agricultural (A) and manufactured (M). Preferences are described by a homothetic, continuous utility function $U(A, M)$. There is a continuum of locations $\ell \in [0, 1]$ on a line segment with unit mass. Endowments are distributed across space according to continuous functions $A(\ell)$ and $M(\ell)$. We assume that the relative endowment $m(\ell) \equiv M(\ell)/A(\ell)$ is increasing so that comparative advantage in manufactures is increasing towards the east.[3]

2.2 Borders and Countries

Space is partitioned into a finite number of $N \geq 1$ intervals by a set of borders such that

$$0 \ = \ b_0 < .. < b_{n-1} < b_n < ... < b_N \ = \ 1.$$

Borders b_0 and b_N are **natural borders**. A **country** is a set of adjacent locations between two consecutive borders b_{n-1} and b_n that can trade with each other. An interval does not constitute a country if it is comprised of autarkic locations. In what follows, we will show that this does not arise in equilibrium. An **endogenous border** b_n with $1 \leq n \leq N-1$ delineates at least one country.

We can label countries as an interval or index them by their ordering. In former case, they can be open (b_{n-1}, b_n), closed $[b_{n-1}, b_n]$ or semi-closed on either side, depending on whether a border belongs to the country or not.

What distinguishes a country is that its locations can trade with each other but not with locations outside of the set.[4] The implicit assumption is that external trade costs are infinite. For

[3]This endowment economy could be generalized to a production economy in which there is linear production, and either population is fixed or there is another spatially fixed factor. For example, we could re-label food and manufactures into farmers and artisans, or we could have have a perfect complement technology with immobile factors, e.g. land and capital).

[4]With this definition of a country, we rule out countries with disjoint regions, such as Alaska for the USA or Kaliningrad for Russia. To be more specific, we rule out disjoint locations being in the same country if trade between the two requires transiting through another country. For the simplistic case, consider the space being a one-dimensional island or a continent (i.e., a large island), and the only means of transportation being overland.

simplicity, we assume that internal trade are zero. That is, countries are autarkic with respect to each other while locations within a country can trade without any frictions.

We make two assumptions that simplify the analysis and help us focus on key insights on how comparative advantages shape the relative size of countries. First,

Assumption 1. *The number of countries is exogenously given.*

This is in contrast to Alesina and Spolaore (1997) who solve for the equilibrium number of countries by restricting them to be symmetrically sized. By relaxing the symmetry assumption, we investigate the stability of any partition of space in greater generality. As in Alesina and Spolaore (1997), one can assume that maintaining a country is convexly costly in its size, which could help endogenize the number of countries. This would, however, significantly complicate the analysis and distract from the key mechanism we wish to highlight.

Second, countries in our setting are akin to coalitions of locations. The relevant equilibrium solution concept depends on whether utility is transferable across locations. We assume that it is not:

Assumption 2. *Utility is non-transferable between locations.*

2.3 Prices and Pattern of Trade

Manufactures are the numeraire in all countries and in all autarkic locations. Call P_n and p_ℓ the equilibrium price of food in country n and location ℓ, respectively. By the assumption of zero internal trade costs, $p_\ell = P_n$ if location ℓ is in country n. Relative demand $D(P)$ for manufactures is increasing in the price of food, i.e., $D'(P) > 0$.

A country delineated by b_{n-1} and b_n has an equilibrium price P_n satisfying the market clearance condition

$$D(P_n) = \frac{\int_{b_{n-1}}^{b_n} M(\ell)d\ell}{\int_{b_{n-1}}^{b_n} A(\ell)d\ell}, \tag{1}$$

where on the left side is relative demand of manufactures and on the right is relative supply. A location ℓ which does not belong to a country has an autarky price p_ℓ^a satisfying $D(p_\ell^a) = m(\ell)$.

Given the assumption $m'(\ell) > 0$, p^a_ℓ is also increasing in ℓ.

The pattern of trade for a location depends on whether it faces a price P_n greater or smaller than its autarky price p^a_ℓ. If $P_n > p^a_\ell$, location ℓ would be a net exporter of food. Otherwise, if $P_n < p^a_\ell$, it would export manufactures in exchange for food. To see how trade patterns align with geography, note that each country n has a unique "price center" C_n, a location defined by $p^a_{C_n} = P_n$. That is, the price prevailing in the country is equal to the autarky price of C_n, making the price center the average location with no comparative advantage. It necessarily satisfies $b_{n-1} < C_n < b_n$. All locations $\ell < C_n$ are net exporters of food, while all $\ell > C_n$ are net exporters of manufactures.[5]

Since the location index also captures endowments, indirect utility of ℓ for an equilibrium price P is given by $v(\ell, P)$. The standard terms of trade effect implies that

$$\frac{\partial v(\ell, P)}{\partial P} > 0 \quad \text{if } P > p^a_\ell,$$

$$\frac{\partial v(\ell, P)}{\partial P} < 0 \quad \text{if } P < p^a_\ell.$$

Intuitively, the utility of a net exporter of food increases with the price of food, while the utility of a net exporter decreases.

Facing its autarky relative price, a price center attains the lowest level of welfare it can in the model: a location would strictly prefer not to be the price center of any country with positive mass to being a price center since

$$v(\ell, P) > v(\ell, p^a_\ell) \quad \text{for any} \quad P \neq p^a_\ell.$$

As implied by the terms of trade mechanism, gains from trade are increasing in the difference between the equilibrium trade price and autarky price. This has a geographical interpretation in our model: that gains from trade for a location increases with its distance to the price center.

[5]Note that the price center does not need to be the geographic center of the country. That would only be the case under specific assumptions on preferences and the distribution of $m(\ell)$. In models of economic geography with fixed factors, geographic centrality may be an advantage if internal trade is costly (Allen and Arkolakis, 2014; Coşar and Fajgelbaum, 2016). In our baseline model with costless internal trade, this is not the case.

Define gains from trade as a share of autarky utility:

$$\pi(\ell, P_n) \; = \; \frac{v(\ell, P_n) - v(\ell, p^a_\ell)}{v(\ell, p^a_\ell)}.$$

Lemma 3 in the Appendix shows that locations that are further away from the price center enjoy greater gains from trade:

$$\frac{\partial \pi(\ell, P_n)}{\partial \ell} \; < \; 0 \quad \text{if } P_n > p^a_\ell,$$

$$\frac{\pi(\ell, P_n)}{\partial \ell} \; > \; 0 \quad \text{if } P_n > p^a_\ell.$$

We now move on to elaborate how endogenous borders can potentially change as a result of secessions and territorial aggregations.

3 Country Formation

This section defines the distinct ways through which endogenous borders may change to form countries and elucidates various cases through examples. Note that we already assumed contiguity as a condition to form a country. Given any initial partition, there are two possible cases through which new countries can come to existence:

Definition 1. *A* ***secession*** *happens when a set of adjacent locations, including only subsets of pre-existing countries, form a new country.*

Definition 2. *An* ***aggregation*** *happens when a set of adjacent locations, including at least one pre-existing country or a set of adjacent autarkic locations, form a new country. Whenever a pre-existing country is involved, an aggregation is* ***center-preserving*** *from its perspective if its price center does not shift. Otherwise, it is* ***center-shifting****.*

Since the definition of aggregation allows for a variety of cases, we proceed by way of examples that help define these typologies.

First, an aggregation happens when a country annexes portions of neighboring intervals. For example, suppose there are initially five intervals, with borders at $0 = b_0 < b_1 < b_2 < b_3 < b_4 < b_5 = 1$. The country $[b_1, b_2]$ could then annex regions $[b_1 - a_l, b_1)$ and $(b_2, b_2 + a_r]$ satisfying $0 \leq a_l < b_1$ and $0 \leq a_r < b_3 - b_2$ with either $0 < a_l$ *or* $0 < a_r$, giving birth to country $[b_1 - a_l, b_2 + a_r]$. An annexation can be center-preserving if both $0 < a_l$ *and* $0 < a_r$, and is necessarily center-shifting if either equals zero.

Second, an atomistic aggregation happens when an individual location, which is adjacent to but not part of a country, joins that country. As an example, suppose that country 2 is initially an open set, (b_1, b_2). The joining of border location b_1 or b_2 to country 2 would be an atomistic aggregation. Atomistic aggregations are center-preserving from the perspective of the country involved.

Third, there is a case in which two or more adjacent countries merge to form a new country. For example, countries $[b_1, b_2]$ and $(b_2, b_3]$ could merge into country $[b_1, b_3]$: from the perspective of $[b_1, b_2]$, this would be an aggregation with $a_l = 0$ and $a_r = b_3 - b_2$, while from the perspective of $(b_2, b_3]$, it would be an aggregation with $a_l = b_2 - b_1$ and $a_r = 0$. Alternatively, countries $[b_1, b_2]$, $(b_2, b_3]$ and $(b_3, b_4]$ could merge into country $[b_1, b_4]$ or countries $[b_1, b_2]$, $(b_2, b_3]$, $(b_3, b_4]$ and $(b_4, b_5]$ could merge into country $(b_1, b_5]$. There is also a case in which two or more adjacent countries merge together, also adding bits at the extremes. For example, this could give birth to country $[b_1, b_4 + a_r]$, with $a_r < b_5 - b_4$. A merger is necessarily center-shifting for at least one of the countries involved and center-preserving for at most one of the countries involved.

Finally, there is a case in which an interval is comprised of a set of adjacent autarkic locations. These can aggregate to form a new country with positive mass, or if there exists a country adjacent to this interval, they can join that country. Note that from the perspective of the pre-existing country, this case has been considered above as an aggregation through annexation.

4 Equilibrium Borders

We are now ready to characterize political equilibria. In our setting, political equilibrium comprises stable coalitions of contiguous locations that leave no incentives for deviation.

Given a border partition $\{b_n\}_{n=1}^{n=N-1}$, any secession or aggregation that results in the formation of a new country requires the consent of all the locations becoming part of the new country. In order words, in the case of a secession, all seceding locations, and in the case of an aggregation, all locations in the proposed expanded country, incumbent or new, have veto power. In the absence of transfers and a technology to dominate other locations, any viable change requires the unanimous consent of all the locations in the newly emerging country.[6] To characterize stability, we thus consider whether there is a *viable* secession or aggregation that makes all locations becoming part of the new country at least as well off as before, with some of them becoming strictly better off. A political equilibrium can be defined as the absence of such deviations:

Definition 3. *A set of borders is in* ***political equilibrium*** *when*

- *there is no secession that makes all seceding locations at least as well off and some strictly better off,*
- *there is no aggregation that makes all locations in newly emerging countries at least as well off and some strictly better off.*

We start the characterization of stability by observing that if some locations are in autarky, a viable aggregation exists and that is the aggregation of such autarkic locations to countries with positive mass. The following lemma states this result formally.

Lemma 1. *No situation in which some location is in autarky is a political equilibrium.*

Proof. If there exists an autarkic location ℓ which is adjacent to some country $[b_{n-1}, \ell)$ or (b_{n-1}, ℓ) to its west (w.l.o.g), such that $0 \leq b_{n-1} < \ell$, by joining this country, location ℓ would move from autarky price p_ℓ^a to $P_n < p_\ell^a$, and thus increase its utility. At the same time, the utility of incumbent locations in the country does not change. Since this constitutes a viable atomistic aggregation, such a location cannot exist in a political equilibrium.

If there exists no autarkic location ℓ which is adjacent to some country, then the entire unit interval is comprised of adjacent autarkic locations. If they aggregate to form a country, any

[6] Other forms of empirically relevant allocation protocols involve majoritarian voting as in Alesina and Spolaore (1997), or domination by force as in Jordan (2006) and its spatial application in Jung (2009).

location that is not the new price center is strictly better off. At the same time, the utility of the price center of the newly aggregated country does not change. Again, since this constitutes a viable aggregation, such locations cannot exist in a political equilibrium. □

Put differently, a completely disconnected world with all locations in isolation is not an equilibrium. Similarly, any autarkic location would be strictly better off by integrating to an adjacent country with positive mass, and all locations in that country would be indifferent. That implies in equilibrium, all endogenous borders delineate countries on both sides. As such, when there is no interval of autarkic locations, the number of countries equals N.

Lemma 2. *Any situation in which no location is in autarky is robust to secession and center-shifting aggregation.*

Proof. Consider secession by a set S of adjacent locations $[s_l, s_h]$ with the price center C_S.[7] If S crosses some existing endogenous border, that is $s_l < b_n < s_h$ for some b_n, the utility of the new price center C_S decreases, since it moves from either P_n or P_{n+1} to autarky price $p^a_{C_S}$.[8] If S does not cross any endogenous border, it must then belong to some country n such that $S \subset [b_{n-1}, b_n]$. If $C_S \neq C_n$, then the utility of C_S decreases. If $C_S = C_n$, the utility of all locations in S remains the same. In all possible cases, the secession is not viable. A center-shifting aggregation is trivially not viable since the new price center would be strictly worse off, and thus block the change of borders. □

Lemma 2 implies that an integrated world $[0, 1]$, i.e., the absence of an endogenous border, constitutes a political equilibrium. To see this, note that the integrated world economy has one price center. Any secession would create at least two new price centers, who would block such change. Thus if $N = 1$, the situation is a political equilibrium.

Taking stock, by Lemma 1 we eliminated the existence of autarkic locations in a political equilibrium. By Lemma 2, we showed that when no location is in autarky, secessions and center-shifting aggregations are not possible under any partition of space into countries. What remains

[7] Whether the initial and seceding countries are closed, open or half-closed intervals does not matter for the argument.

[8] Note that it must be $P_n \neq p^a_{C_S}$ if C_S initially belongs to country n: this is because it is $b_{n-1} \leq s_l$ and $b_n \leq s_h$ (with at least one strict inequality), which implies $C_n < C_S$. By a symmetric logic, it must be $P_{n+1} \neq p^a_{C_S}$ if C_S initially belongs to country $n + 1$.

to be characterized is robustness to center-preserving aggregations involving regions with positive mass. But first, we show that under an intuitive assumption on the country choice of endogenous border locations, the case of two countries becomes straightforward.

4.1 Stability with two countries

So far we used borders b_{n-1} and b_n to delineate a country n without specifying whether countries are open, closed or half-closed intervals, i.e., we did not describe whether border location b_{n-1} belongs to the country $n-1$ to its west or to country n to its east. Since the allegiance of an atomistic location does not affect the prices in these countries, we assume that if a border location strictly prefers to be a part of the country to its west or east, it is able to do so:

Assumption 3. *If $v(b_{n-1}, P_{n-1}) > v(b_{n-1}, P_n)$, then b_{n-1} is part of country $n-1$ to its west. If $v(b_{n-1}, P_{n-1}) < v(b_{n-1}, P_n)$, then b_{n-1} is part of country n to its east.*

This assumption renders the case of two countries straightforward. If the only endogenous border location $0 < b_1 < 1$ is part of the country where it is strictly better off, an atomistic aggregation is not an issue. With just one endogenous border, no other center-preserving aggregation exists. Finally, as proven by Lemma 2, any secession or center-shifting aggregation is not viable. For example, the merger of the two countries into an integrated world would create a new price center that would be strictly off and would thus block the change. Thus if $N = 2$, the situation is also a political equilibrium.

Note that Assumption 3 did not spell out the case of indifference for a border location between the prices of countries surrounding it. Such a case can be resolved through a coin-flip. If the border, however, is not indifferent, there is another location that is. For any two adjacent countries with prices P_{n-1} and P_n, there is a location situated strictly between the price centers, $\ell \in (C_{n-1}, C_n)$, satisfying

$$v(\ell, P_{n-1}) \;=\; v(\ell, P_n).$$

This location receives the same utility when trading at either of the prices prevailing to its west or to its east, constituting a threshold of indifference between the two centers or countries. As alluded above, the threshold may or may not coincide with the actual border b_{n-1}. Since this comparison turns out to be helpful in describing whether an actual endogenous border is stable or not, we introduce the following definition:

Definition 4. *For any partition of space into N countries, implying equilibrium prices $\{P_n\}_{n=1}^{n=N}$, there is a set of* ***indifference thresholds*** $\{\hat{b}_n\}_{n=1}^{n=N-1}$ *such that*

$$v(\hat{b}_n, P_n) \; = \; v(\hat{b}_n, P_{n+1}) \quad \textit{for all } n \in \{1, ..., N-1\}.$$

Note that *indifference* thresholds are implicitly defined by a given set of *actual* borders, and may deviate from them. In what follows, the location of actual borders with respect to indifference borders will play a key role in characterizing their stability. In particular, we will make use of Lemma 3 in the Appendix which implies that given any two countries n and $n+1$, all locations to the east of $\hat{b}_n$ strictly prefer country n. If the endogenous border is situated in that range, i.e., $\hat{b}_n < b_n$, it joins country n. If it is to the west of the indifference threshold, it joins country $n+1$ to the east. We illustrate this in Figure 1 which plots all relevant locations defined so far for the case of $N = 2$. The two countries have price centers C_1 and C_2. Since the endogenous border b_1 lies to the east of the indifference threshold $\hat{b}_1$, it choses to be part of country 1.

4.2 Stability with more than two countries

For the general case of $N > 2$, our main result involves ruling out cases for which center-preserving aggregations are viable:

Proposition 1. *A situation with $N > 2$ in which no location is in autarchy is robust to center-preserving aggregation if and only if there exists no country n with borders b_{n-1} and b_n such that $b_{n-1} \leq \hat{b}_{n-1}$ and $b_n \geq \hat{b}_n$.*

Proof. Any centre-preserving aggregation (other than atomistic aggregations) must result in a country with borders at $b_{n-1} - a_l$ and $b_n + a_r$, where it is $a_l, a_r > 0$ and b_{n-1} and b_n are endogenous borders. To prove sufficiency, suppose first that the condition posed by the proposition is satisfied. Then, either location b_{n-1} prefers to be in country $n-1$ than in country n (and, by Assumption

3, is initially in country $n-1$), or location b_n prefers to be in country $n+1$ than in country n (and, by Assumption 3, is initially in country $n+1$), or both. But for both locations, the aggregation is akin to joining country n, since the newly emerging country has price P_n. Since at least one of the two locations is made worse off by the aggregation, this is not viable. The situation is then a political equilibrium.

To prove necessity, suppose that the condition is not satisfied. Then, there exists a country n such that location b_{n-1} weakly prefers to be in country n than in country $n-1$ and location b_n weakly prefers to be in country n than in country $n+1$. It is always possible to find two numbers $a_l > 0$ and $a_r > 0$ such that the aggregation to form a country with borders at $b_{n-1} - a_l$ and $b_n + a_r$ is center-preserving, location $b_{n-1} - a_l$ is in country $n-1$, and location $b_n + a_r$ is in country $n+1$. By Lemma 3, all locations in $[b_{n-1} - a_l, b_{n-1}]$ prefer to be in country n than in country $n-1$, and all locations in $[b_n, b_n + a_r]$ prefer to be in country n than in country $n+1$. At the same time, all locations in (b_{n-1}, b_n) are indifferent. It follows that the aggregation is viable, and the situation is not a political equilibrium. □

In an equilibrium, it can never be the case that the two border locations which define a country both weakly prefer to belong to that country, since otherwise locations just outside of the country's borders would like to be annexed by it. This poses a restrictions on the maximum size that any country can achieve relative to its neighbours. Intuitively, an excessively large country, having a relatively large domestic market, would make locations situated just outside of its borders want to be annexed by it, since they would then become able to trade with locations whose endowments are, on average more, different from their own. This implies that, in equilibrium, any country must have at least one border such that locations just outside of that border are not interested in being annexed. By continuity, this also implies that locations situated just inside of that border would be better off by being annexed by the neighbouring country, but they cannot because of the impossibility to form a coalition achieving that.

The condition laid out in Proposition 1 requires the preferences of endogenous borders location to be distributed according to one of several possible configurations. Figure 2 gives an example for the case of five countries. In order for the condition to be satisfied, borders must located in such a way that all border locations prefer to belong to the country to their west (panel 1), or they all prefer to belong to the country to their east (panel 2), or there is a country in the middle whose border location both prefer not to belong to it (panel 3).[9] In any other cases (e.g. in the case in panel 4), the condition is violated.

By Lemma 2 and Assumption 3, all situations with $N > 2$ are robust to secessions, centre-shifting aggregation, and atomistic aggregation. Then, the condition laid out in Proposition 1 is also necessary and sufficient for a situation with $N > 2$ to be a political equilibrium.

5 Simulations and Comparative Statics

As discussed above, the stability conditions on borders impose a restriction on the relative size of countries with respect to their neighbours. An natural question is how the relative size of countries varies with comparative advantages and the potential gains from trade. While the conditions laid out by Proposition 1 inform us about this relationship, comparative statics are not possible at the level of generality we presented the theory. Therefore, we proceed numerically in order to demonstrate how heterogeneity in relative endowments is related to the variance of equilibrium country sizes. We restrict attention to the three country case.[10]

In particular, we assume a CES utility function with the elasticity of substitution σ between A and M. We endow all locations with one unit of food, $A(\ell) = 1$, and let the distribution of manufacturers be

$$M(\ell) \;=\; \frac{1}{2} + \alpha_2 \left(\ell - \frac{1}{2} \right).$$

The relative price in country n is now

$$P_n = (1 - \alpha_2)\,\frac{1}{2} + \alpha_2 \frac{b_{n-1} + b_n}{2},$$

[9]There are also many variations of each of these cases, where at most one border location is indifferent.
[10]We verified for a large number of cases that the comparative static exercise to be presented is qualitatively identical when there are more than three countries.

and the indifference thresholds $\hat{b}_n$ are implicitly defined by

$$\frac{P_n + M(\hat{b}_n)}{\left(P_n^{1-\sigma}+1\right)^{\frac{1}{1-\sigma}}} = \frac{P_{n+1} + M(\hat{b}_n)}{\left(P_{n+1}^{1-\sigma}+1\right)^{\frac{1}{1-\sigma}}}.$$

With this specification, α_2 governs comparative advantages. A higher α_2 implies stronger comparative advantage force since locations are then more heterogeneous in terms of relative endowments (Figure 3).

To simulate equilibrium borders, we randomly draw a high number of endogenous border locations (b_1, b_2) from a uniform distribution on the unit line, and check whether they satisfy the conditions of a political equilibrium for various levels of α_2. Isolating the draws satisfying stability conditions defined by Proposition 1, we plot the variance of size across stable countries in Figure 4. Heterogeneity and thus the strength of comparative advantages increases with α_2 in the x-axis.

Note that heterogeneity in endowments shrinks the variance, leading to stable borders between more homogeneously sized countries. Concurrently, the gap between the largest and smallest countries narrows. Less heterogeneity increases the size of the largest country. The intuition is that with a flatter gradient of heterogeneity, regions prefer to be further away from the geographical center of their country in order to gain heterogeneity through distance. The lower the elasticity of substitution (σ) between goods, the more pronounced is the effect since a lower elasticity implies a greater role for trade. When $\sigma \to \infty$ (the flat dot-dashed line in Figure 4), trade doesn't matter and all locations are indifferent to any partition of space. In this case, size variance is dictated by the statistical properties of the uniform distribution used in the simulation.

6 Empirical Evidence

The simulation-based comparative static exercise delivered a clean observable: increased variation in the underlying endowments across regions increases the propensity that more homogenous countries arise in equilibrium. In this section, we undertake a first-pass check on whether this implication comes across in the data.

We use publicly available data from Michalopoulos (2012). The dataset contains a set of subregions for each country, informing us about average agricultural suitability, elevation, and time since agricultural transition for each grid cell.

We first estimate the following relationship for adjacent country pairs i, j

$$\ln(VarSize_{ij}) \;=\; \alpha \ln(VarAgr_{ij}) \;+\; \gamma X_{ij} \;+\; \epsilon_{ij},$$

where the dependent variable is the natural logarithm of the variance of country sizes,

$$VarSize_{ij} \;=\; \left(Size_i - \overline{Size}_{ij}\right)^2 \;+\; \left(Size_j - \overline{Size}_{ij}\right)^2.$$

Here, $Size_i$ is the total area of country i and $\overline{Size}_{ij}$ is the average area of the pair.

The key right-hand side variable $\ln(VarAgr_{ij})$ is the cross-country variance of the within-country standard deviations of agricultural suitability. Specifically, let σ_i^{agr} be the standard deviation of agricultural suitability across the regions of country i, capturing heterogeneity within each country. $VarAgr_{ij}$ is then the pairwise variance of this within-country heterogeneity:

$$VarAgr_{ij} \;=\; \left(\sigma_i^{agr} - \overline{\sigma^{agr}}_{ij}\right)^2 \;+\; \left(\sigma_j^{agr} - \overline{\sigma^{agr}}_{ij}\right)^2.$$

If variation in agricultural suitability is a good measure for capturing the variation in relative endowments, and thus comparative advantages, we would expect α to be negative.

X_{ij} are control variables. Continent dummies are intuitive. Mean latitude of the country pair and its interaction with $VarSize_{ij}$ controls for the possibility that the relationship may vary by climatic zones. We also control for the difference between country pairs in their average agricultural suitability, $\ln\left(\left|\overline{Agr}_i - \overline{Agr}_j\right|\right)$. The idea is that a disproportionately fertile and resourceful country may dominate its neighbor, creating a size asymmetry. For the same reason, we also control for the difference in the years since the transition of the country's region to the neolithic age, i.e., since the agricultural revolution. The results in the table are robust to including other controls (not reported) such as colonial/past occupation relationship, difference in number of languages per area and same language dummy.

Tables 1 and 2 present the results for country pairs and triplets, respectively. These preliminary estimates show that the variance of size is significantly and negatively correlated with the variance of agricultural suitability once latitude is controlled for, as implied by our theory.

7 Concluding Remarks

It is worth finishing by highlighting the key difference between our model and that of Alesina and Spolaore (1997). In their case, size benefits a location through increasing returns. On the other side of the trade-off, utility decreases with distance to the center in their case because of heterogeneity of preferences. In our model, there are no intrinsic differences in preferences related to location. Rather, size implies distance for peripheral locations. Distance implies higher difference from the average, thus stronger comparative advantage, and higher gains from trade within that country. While we cannot characterize the equilibrium number of countries, our distinct and novel trade channel allows us to analyze a much larger set of size asymmetries between countries. No doubt that both of these forces are operating in the real world: more distant places are likely to diverge in their preferences and are thus less likely to agree on public policies, they are also more likely to diverge in comparative advantages are thus more likely to gain from trading with each other. The resolution of this tension is a key determinant of territorial configurations called countries.

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Appendix

Lemma 3. *Consider any two locations h and k belonging to country n. Without loss of generality, suppose that it is either $p^a_h < p^a_k < P_n$ or $p^a_h > p^a_k > P_n$. Then, if preferences are homothetic, gains from trade as a share of autarky utility are greater in h than in k,*

$$\pi(h, P_n) > \pi(k, P_n),$$

Proof. Let $M(\ell, P_n)$ and $A(\ell, P_n)$ denote demand by location ℓ when facing price P_n. Consider any (possibly fictitious) location h' such that $A(h') = \alpha_h A(h)$ and $M(h') = \alpha_h M(h)$ for some $\alpha_h > 0$. By the homotheticity of preferences, it must be $p^a_h = p^a_l$. It must also be $M(h', P_n) = \alpha_h M(h, P_n)$ and $A(h', P_n) = \alpha_h A(h, P_n)$. To see this, note that by homotheticity of preferences, it must be $M(h', P_n) = \beta M(h, P_n)$ and $A(h', P_n) = \beta A(h, P_n)$ for some $\beta > 0$. But we can write

$$\begin{aligned}
P_n A(h', P_n) + M(h', P_n) &= P_n A(h') + M(h') \\
P_n \beta A(h|P_n) + \beta M(h|P_n) &= P_n \alpha_h A(h) + \alpha_h M(h) \\
\beta[P_n A(h, P_n) + M(h, P_n)] &= \alpha_h [P_n A(h) + M(h)] \\
&= \alpha_h [P_n A(h, P_n) + M(h, P_n)],
\end{aligned}$$

which immediately implies $\beta = \alpha_h$. By homogeneity of the utility function, it is then $v(h', p^a_{h'}) = \alpha_h v(h, p^a_h)$ and $v(h', P_n) = \alpha_h v(h, P_n)$, which implies $\pi(h', P_n) = \pi(h, P_n)$. By the same logic, for any (possibly fictitious) location k' such that $A(k') = \alpha_k A(k)$ and $M(k') = \alpha_k M(k)$ for some $\alpha_k > 0$, it must be $p^a_{k'} = p^a_k$, and $\pi(k', P_n) = \pi(k, P_n)$.

Suppose now that we chose α_h and α_k so that h' and k' have the same income at price P_n (such a choice is always available). Since $p^a_h < p^a_k < P_n$, it is $p^a_{h'} < p^a_{k'} < P_n$, which by homotheticity implies $M(h')/A(h') < M(k')/A(k')$. Consider the autarchy income of location k'. This is

$$\begin{aligned}
p^a_{k'} A(k') + M(k') &= p^a_{k'} \left[A(k') + \frac{M(k') - M(h')}{p^a_{k'}}\right] + M(h') \\
&> p^a_{k'} [A(k') + A(h') - A(k')] + M(h') \\
&= p^a_{k'} A(h') + M(h'),
\end{aligned}$$

where the inequality follows from the fact that

$$\begin{aligned} P_n A(h') + M(h') &= P_n A(k') + M(k') \\ A(h') - A(k') &= \frac{M(k') - A(h')}{P_n} \\ &< \frac{M(k') - M(h')}{p^a_{k'}} \end{aligned}$$

Since the bundle $(A(h'), M(h'))$ is affordable to location k' in autarchy, but not chosen, it must give lower utility than bundle $(A(k'), M(k'))$, which implies $v(h', p^a_{h'}) < v(k', p^a_{k'})$. At the same time, since h' and k' have the same income when trading at price P_n, it must be $v(h', P_n) = v(k', P_n)$. This implies $\pi(h', P_n) > \pi(k', P_n)$, which in turn implies $\pi(h, P_n) > \pi(k, P_n)$.

The proof for the case $p^a_h > p^a_k > P_n$ proceeds symmetrically. □

Figures

Figure 1: An Example with Two Countries

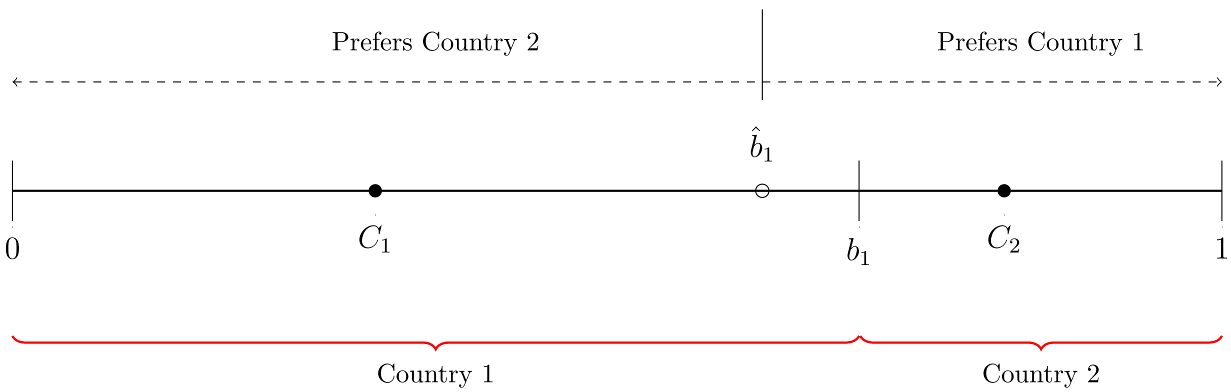


Figure 2: Illustration of Stable and Non-Stable Endogenous Borders

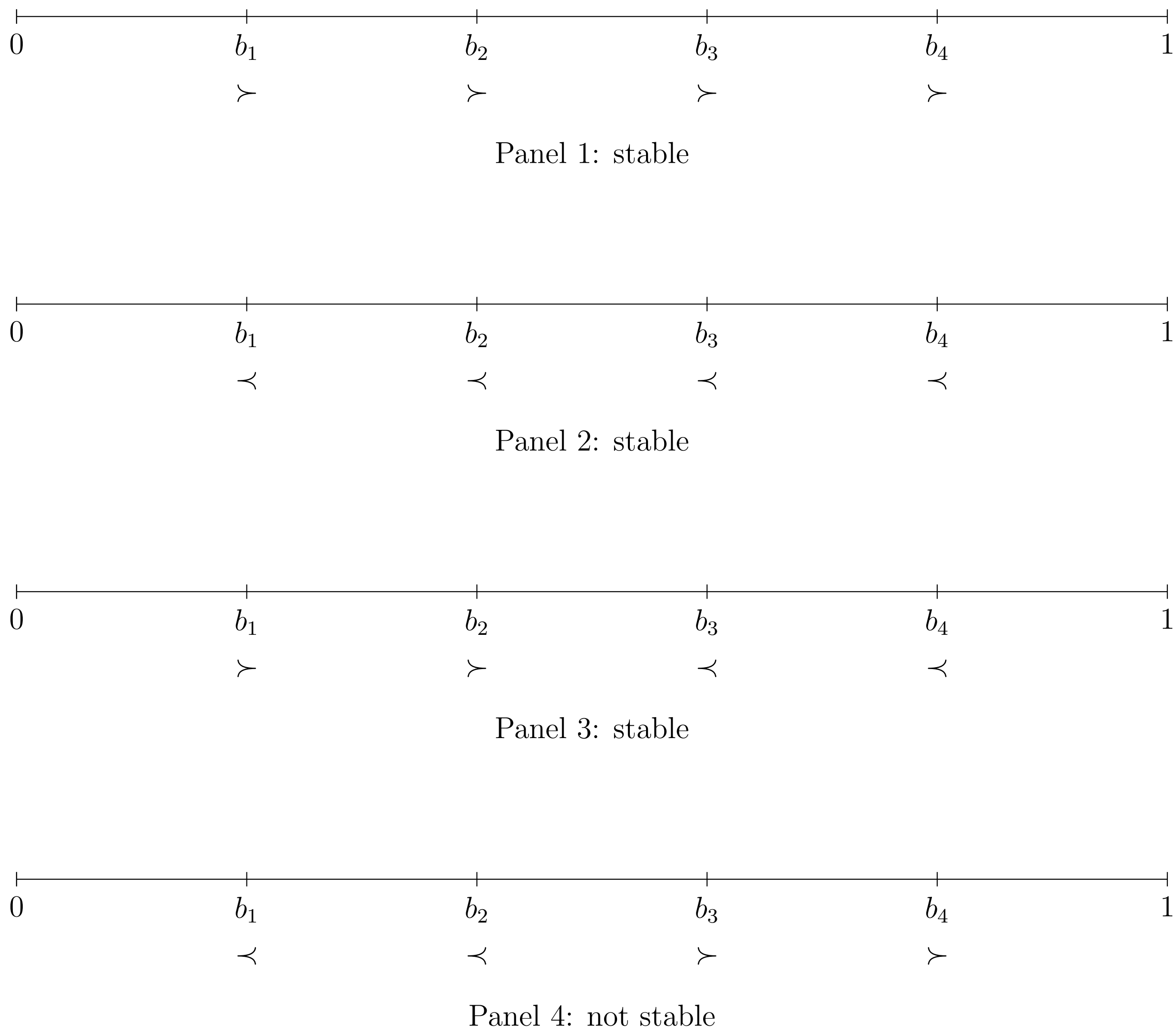
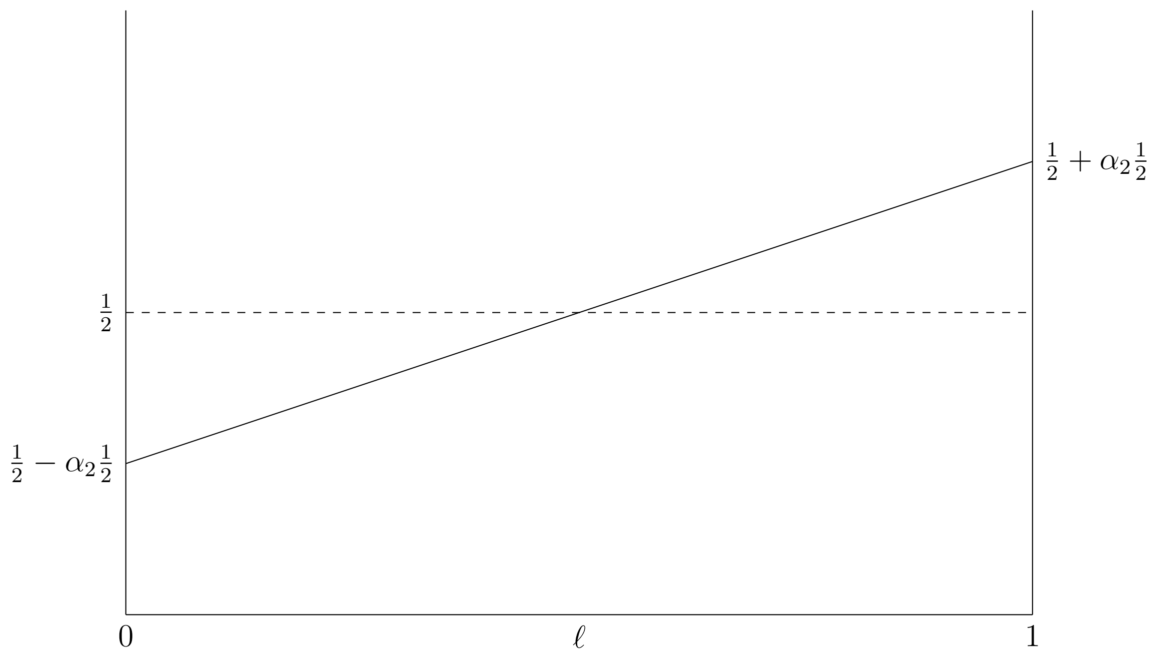
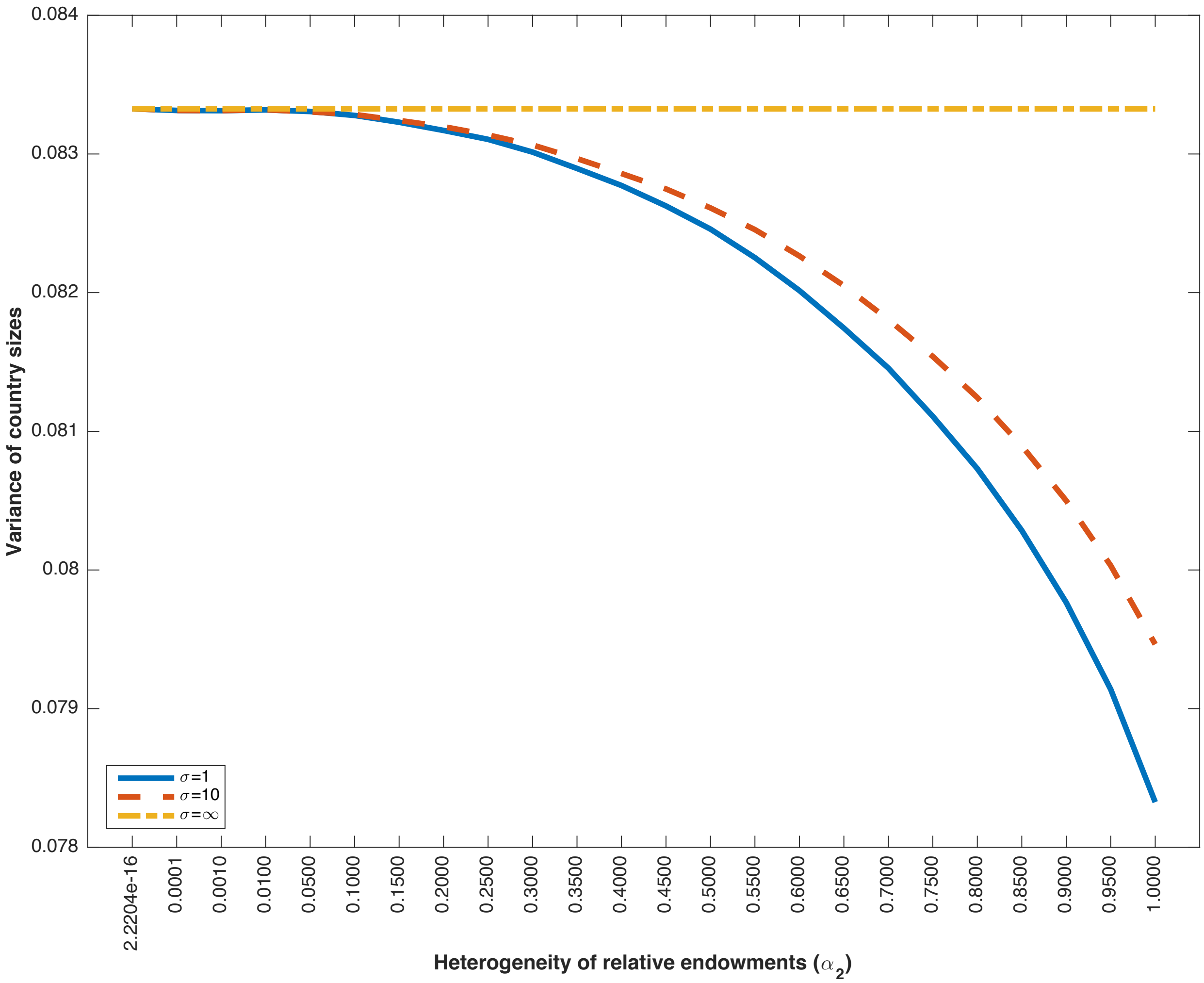


Figure 3: Distribution of Manufacturing (M) Endowments for the Simulated Case



Notes: The solid line depicts the parameterized linear distribution of manufactures across space given by $M(\ell) = \frac{1}{2} + \alpha_2(\ell - \frac{1}{2})$.

Figure 4: Comparative Advantage and Variance of Country Size with three Countries



Notes: In the axis, α_2 controls the gradient of relative endowments and heterogeneity across locations. A higher α_2 implies higher gains from trade between any two locations. All lines are averages across cases that satisfy equilibrium conditions from a random draw of one million border configurations.

Tables

Table 1: Ecological Heterogeneity and Size Variance across Adjacent Country Pairs

Dependent variable $\ln(VarSize_{ij})$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\ln(VarAgrYield_{ij})$	0.200	-0.0232	-2.640***	-2.544***	-1.631***	-1.779***	-1.888***
	(0.187)	(0.229)	(0.180)	(0.121)	(0.399)	(0.411)	(0.377)
$\ln(DifAgrYield_{ij})$		0.204*	0.0753				
		(0.112)	(0.113)				
$\overline{Latitude}_{ij}$			0.191***	0.190***	0.145***	0.152***	0.148***
			(0.0105)	(0.0104)	(0.0260)	(0.0262)	(0.0246)
$\overline{Latitude}_{ij} \times \ln(VarAgrYield_{ij})$			0.106***	0.106***	0.0734***	0.0779***	0.0787***
			(0.00662)	(0.00661)	(0.0146)	(0.0148)	(0.0137)
$\ln(DifAgrTime_{ij})$							0.465***
							(0.107)
$Africa_{ij}$	4.806***	4.848***			1.456**	1.175*	1.540**
	(0.328)	(0.323)			(0.686)	(0.697)	(0.648)
$Americas_{ij}$	5.630***	5.761***			2.230***	1.995***	2.484***
	(0.418)	(0.423)			(0.737)	(0.748)	(0.706)
$Eurosia_{ij}$	5.335***	5.450***			1.135	0.880	1.101
	(0.375)	(0.380)			(0.795)	(0.806)	(0.747)
N	264	264	264	264	264	258	258
R^2	0.866	0.867	0.877	0.877	0.884	0.884	0.893

Standard errors in parentheses .

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

$\ln(VarSize_{ij})$: size variance between adjacent countries i, j

$\ln(VarAgrYield_{ij})$: variance of agricultural suitability across regions of adjacent countries i, j

$\ln(DifAgrYield_{ij})$: difference of average agricultural suitability between adjacent countries i, j

$\overline{Latitude}_{ij}$: average latitude (in absolute value) of adjacent countries i, j

$\ln(DifAgrTime_{ij})$: (absolute) difference in years since transition to agriculture between adjacent countries i, j

Continent dummies: equal to one if both countries i, j are in the continent, zero otherwise.

Table 2: Ecological Heterogeneity and Size Variance across Adjacent Country Triplets

Dependent variable $\ln(VarSize_{ijk})$	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\ln(VarAgrYield_{ijk})$	0.525***	0.0829	-3.360***	-3.141***	-1.759***	-1.759***	-2.142***
	(0.103)	(0.129)	(0.0905)	(0.0674)	(0.302)	(0.302)	(0.270)
$\ln(DifAgrYield_{ij})$		0.408***	0.216***		0.223***	0.223***	0.245***
		(0.0740)	(0.0684)		(0.0674)	(0.0674)	(0.0655)
$\overline{Latitude}_{ijk}$			0.222***	0.223***	0.150***	0.150***	0.159***
			(0.00563)	(0.00549)	(0.0177)	(0.0177)	(0.0151)
$\overline{Latitude}_{ijk} \times \ln(VarAgrYield_{ij})$			0.133***	0.133***	0.0738***	0.0738***	0.0809***
			(0.00477)	(0.00469)	(0.0117)	(0.0117)	(0.0100)
$\ln(DifAgrTime_{ijk})$							0.590***
							(0.0522)
$Africa_{ijk}$	5.881***	5.750***			2.146***	2.146***	1.810***
	(0.159)	(0.163)			(0.447)	(0.447)	(0.396)
$Americas_{ijk}$	7.047***	7.059***			3.420***	3.420***	3.338***
	(0.191)	(0.191)			(0.470)	(0.470)	(0.414)
$Eurosia_{ijk}$	6.775***	6.719***			1.754***	1.754***	0.979**
	(0.167)	(0.170)			(0.490)	(0.490)	(0.430)
N	815	815	815	815	815	815	815
R^2	0.945	0.947	0.947	0.947	0.954	0.954	0.961

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

$\ln(VarSize_{ijk})$: size variance between adjacent countries i, j

$\ln(VarAgrYield_{ijk})$: variance of agricultural suitability across regions of adjacent countries i, j, k

$\ln(DifAgrYield_{ijk})$: maximum difference of average agricultural suitability between adjacent countries i, j, k

$\overline{Latitude}_{ijk}$: average latitude (in absolute value) of adjacent countries i, j, k

$\ln(DifAgrTime_{ijk})$: (absolute) maximum difference in years since transition to agriculture between adjacent countries i, j, k

Continent dummies: equal to one if all countries i, j, k are in the continent, zero otherwise.